Rejection Sampling

- Find q(y) that is easy to sample from.
- Find M such that \( \frac{f(x)}{Mq(y)} \leq 1 \)

\[
P(y=1|x) = \frac{P(y=1)}{Mq(y)} \quad \text{Probability of accepting sample}
\]

What's the distribution of the accepted samples?

\[
P(x|y=1) = \frac{P(x|y=1)q(y)}{P(y=1)} = \frac{P(x)}{Mq(y)} \quad \text{Accept sample}
\]

Why stationary distribution of Gibbs Sampling is the distribution?

Given sample \( x^i \) at 1st iteration, we draw the next sample from this distribution

\[
Q(x | x^i) = \sum_i q(i) \frac{P(x_i | x^i)}{\prod_j P(x_j)}
\]

1. First choose one of the variables randomly with prob \( q(i) \)
2. Make sure all other variables are fixed at the previous iteration
3. Sample from conditional

Let's show the stationary distribution is the same.

\[
\int Q(x | x^i) P(x) \, dx = \left( \sum_i q(i) \right) \int P(x_i | x^i) \frac{\prod_j P(x_j)}{P(x^i)} \, dx
\]

\[
= \sum_i q(i) \frac{P(x_i | x^i)}{P(x_i)} \int P(x_i) \frac{\prod_j P(x_j)}{P(x^i)} \, dx
\]

\[
= \sum_i q(i) P(x_i | x^i) \int \frac{P(x)}{P(x_i)} \, dx = \text{P(x)}
\]

\( P(x) \) is a stationary distribution!